Many Uses for Diffusion Waves

I was excited to see "Diffusion Waves and Their Uses" on the cover of the August 2000 issue of PHYSICS TODAY and read Andreas Mandelis's report (page 29) with great anticipation. It is a timely follow-on to an excellent article by Arjun Yodh and Britton Chance ("Spectroscopy and Imaging in Diffusing Light," PHYSICS TODAY, March 1995, page 34) emphasizing optical tomography. A rapidly developing application in atmospheric science where the objects of study are dense clouds provides a further demonstration of the power of diffusing light.

From 1994 through 1996, I was part of a team at NASA's Goddard Space Flight Center working on an observational curiosity in cloud remote sensing using reflected sunlight. The definitive explanation of this curiosity required radiative Green function theory.[1] Ignoring practical considerations, we theoreticians realized that one should go from passive to active remote sensing: A cloud's Green function for photon diffusion could, in principle, be observed directly using laser radar (lidar) techniques. From this diffuse laser photon field, we can reliably (and some day cost-effectively) infer cloud thickness and density, possibly with some information about internal variability, at scales of about 0.5 km.[2]

The conceptual leap was to drop the routine single-scattering model for lidar signals and to work instead with the diffusion equation to model the time-dependent multiple scattering processes inside the opaque cloud mass. The corresponding instrumental leap was to detect the weak "off-beam" signal emerging from the cloud at large distances from the laser beam. This was successfully done under heavily overcast skies at Goddard by September 1996 with a conventional zenith-pointing detector and a near-IR beam gradually deflected away from zenith.[2] The continuous-wave signal, within its natural variability, followed the theoretical prediction out to 12 degrees, corresponding to 0.3 km at cloud base; at that point, it was lost in the solar noise. Now, at NASA there is no progress without an acronym; thus "cloud THickness from Off-beam Returns (THOR) lidar" was born!

In September 1994, the first-ever atmospheric lidar experiment from space, the Lidar-In-space Technological Experiment (LITE), was successfully conducted by NASA's Langley Research Center. LITE pulses returned from dense clouds proved to be temporal counterparts of the spatial Green functions we were to detect at Goddard. In spite of LITE's conventionally narrow field of view (FOV), this is truly an off-beam signal since the unprecedented distance puts essentially all orders of in-cloud scattering into the FOV.

The remaining challenge in THOR lidar is to measure the diffuse laser-generated light field at once in both space and time.

During the same period in the early 1990s, but for entirely different reasons, researchers at Los Alamos National Laboratory had developed a highly sensitive imaging/timing detector for the Remote Ultra-Low Light Imaging (RULLI) project.[3] A colleague of these individuals, Steven Love, quickly recognized that RULLI could be adapted to cloud studies. He and I discussed this at a meeting in 1995. In 1997, I joined the space and remote sensing sciences group at Los Alamos. With help from the RULLI team, we soon fielded a prototype Wide-Angle Imaging Lidar (WAIL). Interested readers can actually see cloud Green functions—literally, for bright green light at 532 nm—in "movies" downloadable from the Web at http://nis-www.lanl.gov/~love/clouds.html.

At Los Alamos we are currently redesigning the source-filter-detector suite to give WAIL daytime capability, hence the ability to monitor the important diurnal cycle of clouds. A parallel effort is under way at Goddard's THOR lab to build an airborne instrument. It will have a massive fiberoptic bundle feed at its focal plane to deliver light to an array of photomultiplier tubes, each assigned to an annular sector around the beam.

It is now easy to see why Yodh and Chance's 1995 article made a deep impression on me. In medical imaging, it is vastly preferable to probe soft tissue with harmless near-IR photons rather than with high levels of x rays, especially to discover that the tissue is healthy. After heavy x-ray exposure, who can be sure? In meteorology, we have good reasons to probe clouds with visible light rather than the currently fashionable microwave approach. Earth's climate is largely controlled by how clouds interact with sunlight; by using green light, we avoid the uncertain translation from the cloud's radar reflectivity to the desired optical properties in the solar spectrum. So, we had invested our limited start-up funds to study "photon migration" and perform "optical tomography" of sorts with clouds, even before learning these expressions.

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The article on diffusion waves by Andreas Mandelis was a refreshing and interesting look at some applications of these waves, which are governed by a linear, parabolic partial differential equation. Readers may be interested in some other applications that arise from extensions of this governing equation in classical continuum mechanics. In porous media transport, for example,

continued on page 100

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continued from page 15

diffusion wave theories have variously been used to describe propagation in poroelastic media,[1] the wave-driven dynamics of reacting solutes in homogeneous media[2] and in mixed media characterized by composite heterogeneity, or by interfaces at which discontinuities in the wave field are maintained.[3]

In the related field of groundwater hydrology, the wave field is identified with hydraulic head and is typically forced by tidal effects in surface water bodies, by Earth tides or by seasonal effects. The governing equation ensures that high frequency modes are strongly damped in aquifers, while low frequency modes are passed.

Of particular interest are nonlinear diffusion waves, which are often relevant for groundwaters near beaches, river banks, and so forth, and which display unusual characteristics.[4] Various examples of nonlinear diffusion waves are described in refs. 4 and 5.

Mandelis concentrated his discussion on applications close to modern physicists' hearts, including materials science, photonics, and so forth. What I hope to have shown is that these peculiar waves are also important in other areas, reinforcing Mandelis's conclusion that diffusive propagation is a topic worthy of further study.

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Andreas Mandelis, in his interesting and informative article, remarks that only recently has significant progress in the subject occurred since the days of Anders Jonas Ångström. Not so! The neutron physics community has known and enjoyed "neutron waves" for the past half-century. (These are not the de Broglie waves associated with the particles but wavelike disturbances in a diffusing cloud of neutrons.) A description of the experiments with neutrons reached textbook-level in 1958. In their classic work, Alvin Weinberg and Eugene Wigner[1] wrote, "This method of measuring the diffusion coefficient in a medium is analogous to Ångström's cyclic method of measuring thermal conductivity."

The elaborate experiments described by Mandelis are difficult, and the very simple model of transport used to analyze them appears acceptable, generally. But it will soon be necessary to go deeper. The proper description of these "waves" follows from a transport equation for the particles; the classical diffusion equation is but a crude approximation of the transport equation. For example, Mandelis notes the spurious instantaneous propagation of disturbances predicted by the diffusion equation. He and his colleagues know that a minor improvement gives the telegrapher's equation, whose solutions propagate with finite speed.

The neutron physics community has spent decades walking the path from diffusion to transport, elucidating the effects of strong absorption, anisotropic scattering, and boundary effects in space and time.[2] Many of us are saddened to find our colleagues in the field of diffusion waves, particularly those dealing with optical tomography,[3] reinventing the wheel by grappling with these issues. Visiting a neighbor's garden can be very productive.

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MANDELIS REPLIES: I was delighted to see the reactions to my article "Diffusion Waves and Their Uses" (Physics Today, August 2000, page 29). In addition to the three letters published here, I received several letters and comments privately. The spectrum of disciplines that encompass diffusion waves is much broader than my article indicates. The article acted as an awareness call to the scientific community regarding the strong interdisciplinary character and analytical–diagnostic potential of diffusion waves.

As Michael Trefry's letter describes, the very recent use of diffusion-wave methods in the study of organic compound migration in stratified media can clarify the transport of cyclic diffusive transients subject to sinusoidal boundary conditions (his ref. 3). To my knowledge, the earliest applications of diffusion waves to mass transport with modulated input sources concern metals, electrolytes, dialysis membranes, and the study of harmonic atomic and molecular diffusion processes through polymers by means of pressure oscillations inside a vacuum chamber.[1] Oscillatory sorption measurements were reported even earlier.[2]

Noel Corngold pointed out the very early work on neutron waves. These are harmonic fluxes of neutrons produced periodically in beryllium surrounded by heavy water (Corngold's ref. 1, p. 212). Many similar harmonic physical phenomena can be described as diffusion waves.

Viscosity waves are still controversial,[3] but there is growing evidence that they are forced into existence by gravity-wave nonlinearities and reflections. Wavelengths can be 8–26 m. At the other extreme, thermal waves at high modulation frequencies can exhibit wavelengths of only a few microns.

The correspondence shows, however, that many scientists consider time-domain diffusive transport as "waves." This is widespread in the literature, including the interesting cloud remote-sensing research based on radiative Green-function theory in the diffusion limit of the transport equation, reported by Anthony Davis (see his ref. 2). I object to this practice, since the "wave" label, mainly describing hyperbolic propagation (for example, D'Alembert traveling waves), is obviously inconsistent with the parabolic nature of diffusion. To be sure, diffusion waves, as the harmonic versions of the diffusion equation in the Fourier transform sense, qualify only as pseudowaves with many shortcomings of the time-domain equations.[4] The spectral decomposition of the time-domain hyperbolic equations with a diffusion term has been used in the study of wave propagation in poroelastic media, and many features of purely diffusion-wave behavior, such as strong spatial damping of the wave amplitude, have been noted, as in Trefry's ref. 1.

The equivalent mathematical underpinnings between time-dependent diffusion and harmonic diffusion-wave equations tend to be glossed over by many researchers who assign to the latter periodic disturbances properties of propagating (hyperbolic) wave fields that properly belong to the former. Corngold's argument that the telegrapher's equation (otherwise known as "second sound,"[5]) is an improvement over the infinitely fast diffusive propagation is correct; however, this has seen little experimental use within the (harmonic) diffusion-wave communities. I guess the main reasons are that the telegrapher's equation is an ad hoc generalization of Fourier's linear diffusion equation, and that there is no real problem in interpreting data by means of simpler diffusion-wave equations satisfying simultaneity rather than causality; and that the time-delayed expressions introduce relaxation times that are short compared to, say, conduction heat transfer times. This large time-scale difference between conductive transport and second-sound-type relaxation time tends to minimize any perceptible differences between instantaneous and time-delayed responses, offering mostly imperceptible exactitude at the expense of additional mathematical complexity.

I agree with Corngold that the introduction of the concept of neutron waves in the 1950s preceded the tremendous growth of diffusion-wave applications in, for example, the photoacoustic and photothermal communities in the last quarter century. Nevertheless, for a primarily experimental field, neutron waves do not seem to have been as important to neutron diffusion science as their more conventional time-resolved counterparts (Corngold's ref. 1 and ref. 2 chap. 4.2). My bias against counting time-resolved diffusion as "diffusion waves" has led me to conclude that only recently has progress occurred in the diffusion-wave area, to which Corngold has objected. I am grateful, however, that he insightfully pointed out the need for more sophisticated analytical approaches to diffusion-wave applications as they spread across many disciplines, such as charge-carrier-wave dynamics and diffuse photon density waves. I agree with his exhortations for cross-fertilization between current diffusion-wave groups and workers in the broader transport physics areas. To date, the opportunity for cross-fertilization remains largely unexplored, exciting, and potentially fruitful territory, especially in the limiting case in which periodic diffusion lengths become commensurate with mean free paths of random microscopic and mesoscopic motion.[6,7]

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Szilard's Inventions Patently Halted

Valentine L. Telegdi wonders (PHYSICS TODAY, October 2000, page 25) why Leo Szilard abandoned his early patent applications on the linear accelerator, cyclotron, betatron and synchrocyclotron. Perhaps, he suggests, Szilard "lost interest in pursuing them," or the patent examiners "may have raised questions of novelty" if they knew the work of Gustaf Ising or others.

This has been one of the mysteries of Szilard's life. Why did he seemingly abandon so many astonishing and career-making inventions? Was it erratic and eccentric behavior, as is usually assumed?

In my talk at the Szilard Centenary in Budapest in 1998,[1] I argued just the opposite—that Szilard was a logical and determined man who has been misjudged. I published an account of his refrigeration inventions with Albert Einstein,[2] and I am grateful to Telegdi for this occasion to discuss Szilard's accelerators.

The German patent examiner's response to Szilard's 1928 application on the linear accelerator still exists. Szilard gave a copy to Einstein, and it is preserved in the Einstein Archives. The examiner rejected the invention as unpatentable with this classic statement:

> Patents can be given only for inventions that permit a commercial use. However, the submitted procedure apparently has only a scientific value. Whether, in accordance with the invention, any commercially useful material can be produced by accelerating artificially-produced positively-charged corpuscles, appears from our present knowledge ruled out. In the whole application, no hint is found that the applicant has produced, or can produce, such material. Obviously the yield would be so tiny, as with atomic disintegration from the natural alpha rays of radioactive substances, that even in the future the prospect of using the invention in commerce has the highest degree of improbability.[3]

Priceless! What was Szilard to do? To prove the patent office wrong, he needed to build the devices. But without a patent, what company would support such a project? Szilard turned to his friend Dennis Gabor, as Szilard recalled in an unpublished letter:

> It was my intention to build some of the machines and I turned over my patent applications to a colleague, Dr. D. Gabor, who at that time was with the Siemens Company and who thought that he might enlist the support of that company for this task. Nothing came of this, however.[4]

Szilard could have stopped there, but he did not. Telegdi notes that in 1934, after fleeing Germany, Szilard filed an application in the UK on betatron and synchrocyclotron designs that were even more sophisticated. Telegdi suggests that this was Szilard's last work on accelerators, but that is not so.

At Oxford University, while searching for an element that might sustain a nuclear chain reaction, Szilard collaborated with James Tuck to build such a betatron. Frederick Lindemann, director of Oxford's Clarendon Laboratory, agreed to fund betatron construction, and plans were moving forward when history intervened. Donald Kerst, who built the first successful betatron, later called