ICPPP 18



# Quantitative Carrier Density Wave Imaging in Silicon Solar Cells Using Photocarrier Radiometry and Lock-in Carrierography

Q. M. Sun<sup>1</sup>  $\cdot$  A. Melnikov<sup>2</sup>  $\cdot$  A. Mandelis<sup>1,2</sup>

Received: 9 October 2015 / Accepted: 22 February 2016 / Published online: 3 March 2016 © Springer Science+Business Media New York 2016

**Abstract** InGaAs camera-based low-frequency homodyne and high-frequency heterodyne lock-in carrierographies (LIC) are introduced for spatially resolved imaging of optoelectronic properties of Si solar cells. Based on the full theory of solar cell photocarrier radiometry (PCR), several simplification steps were performed aiming at the open circuit case, and a concise expression of the base minority carrier density depth profile was obtained. The model shows that solar cell PCR/LIC signals are mainly sensitive to the base minority carrier lifetime. Both homodyne and heterodyne frequency response data at selected locations on a mc-Si solar cell were used to extract the local base minority carrier lifetimes by best fitting local experimental data to theory.

**Keywords** Frequency-domain photoluminescence · Lock-in carrierography (LIC) · Minority carrier lifetime · Photocarrier radiometry (PCR) · Si solar cell

# **1** Introduction

Photocarrier radiometry (PCR) [1], a form of dynamic frequency-domain photoluminescence (PL), spectrally gated to eliminate thermal-infrared photon emissions,

A. Melnikov melnikov@mie.utoronto.ca

<sup>2</sup> Center for Advanced Diffusion-Wave Technologies (CADIFT), Mechanical and Industrial Engineering, University of Toronto, Toronto M5S 3G8, Canada

This article is part of the selected papers presented at the 18th International Conference on Photoacoustic and Photothermal Phenomena.

A. Mandelis mandelis@mie.utoronto.ca

<sup>&</sup>lt;sup>1</sup> School of Optoelectronic Information, University of Electronic Science and Technology of China, Chengdu 610054, China

has proven to be an effective quantitative methodology for carrier transport parameter determination [2,3] and carrier density wave (CDW) imaging [4,5] of Si wafers and solar cells. Lock-in carrierography (LIC) [6], an InGaAs camera-based imaging extension of PCR, has been introduced and developed for quantitative effective carrier lifetime imaging of Si wafers [7], qualitative CDW imaging of Si solar cells [6,8], and contactless measurements of electrical parameters and current-voltage characteristics of Si solar cells [9,10]. Recently, a heterodyne imaging scheme [8,11] was introduced to overcome the speed limitations of InGaAs cameras and realize high-frequency LIC imaging, which allows the frequency-scan range of the camera LIC system to become comparable to the single-element detector PCR system.

This work focuses on the quantitative evaluation of carrier transport parameters of mc-Si solar cells from homodyne PCR data and heterodyne LIC images, using solar cell PCR/LIC theory. Based on the theory of p-n junction ac photothermal response [12], a full model of solar cell PCR was recently established which takes electrical circuit conditions into account [13]. However, model simplification is also valuable because (1) the uniqueness problem in multi-parameter fitting becomes serious due to the large number of unknown parameters involved in the full model, and it is not apparent which parameters of solar cells are sensitive to PCR/LIC signals and which are not and (2) data fitting using the full model takes too long, which restricts its application in the case of quantitative camera LIC imaging in the field, as more than 100 000 pixel-frequency data need to be processed. Therefore, a simplified model aiming at the open circuit condition and spread beam illumination is presented. Although the generality is compromised somewhat, it can be seen that the expressions of the carrier density depth profile and the PCR/LIC signal are very simple and clear, with good accuracy compared to the full model. Carrier bulk lifetimes of a mc-Si solar cell at selected locations were extracted from both homodyne single-element detector data and heterodyne camera images.

#### 2 Theory

The solar cell geometry is assumed to be an abrupt p-n junction fabricated on a p-type substrate wafer (base) using an n-type impurity to form a thin n-type surface layer (emitter). We consider the case that the excitation light is incident on the z = -d plane (d is the junction depth), under the low injection condition (photocarrier density is much lower than the equilibrium majority carrier density). The excess minority carrier densities are governed by

$$\begin{cases} D_{p} \frac{\partial^{2} p}{\partial z^{2}} - \frac{p}{\tau_{p}} - \frac{\partial p}{\partial t} = -G(z, t), \ -d \leq z < 0\\ D_{n} \frac{\partial^{2} n}{\partial z^{2}} - \frac{n}{\tau_{n}} - \frac{\partial n}{\partial t} = -G(z, t), \ z \geq 0. \end{cases}$$
(1)

Here *D* is the minority carrier diffusivity,  $\tau$  the minority carrier lifetime, the subscripts n,p denote electron and hole, respectively, and *G* is the generation rate. Under monochromatic and harmonically modulated excitation, the generation rate is

$$G(z,t) = \frac{I_0(1-R)\beta\eta}{2Dh\nu} e^{-\beta(z+d)} (1+e^{i\omega t}).$$
 (2)

Here  $I_0/2$  is the average excitation intensity, *R* the surface reflectivity,  $\beta$  the optical absorption coefficient,  $\eta$  the quantum efficiency,  $h\nu$  the photon energy, and  $\omega$  the modulation angular frequency. The dc term in Eq. 2 will generate a dc photovoltage  $V_{\rm dc}$  and a dc generation current  $I_{\rm g}^{\rm dc}$ , while the ac term corresponds to the carrier density wave as well as the modulated photovoltage  $V_{\rm ac}$  and photocurrent  $I_{\rm g}^{\rm ac}$ .

The general solutions of Eq. 1 are

$$p(\omega, z) = A_1 \cosh[\sigma_p(\omega)z] + B_1 \sinh[\sigma_p(\omega)z] - C_p e^{-\beta(z+d)}$$
(3a)

$$n(\omega, z) = A_2 e^{\sigma_n(\omega)z} + B_2 e^{-\sigma_n(\omega)z} - C_n e^{-\beta(z+d)}$$
(3b)

with

$$\sigma_{n,p}(\omega) = \sqrt{\frac{1 + i\omega\tau_{n,p}}{D_{n,p}\tau_{n,p}}}, \ C_{n,p} = \frac{I_0(1 - R)\beta\eta}{2D_{n,p}h\nu(\beta^2 - \sigma_{n,p}^2)}.$$
 (3c)

Here the dc solution has been incorporated in Eq. 3 by setting  $\omega = 0$ . The four constants  $A_1$ ,  $B_1$ ,  $A_2$ , and  $B_2$  can be determined from the boundary conditions. For mc-Si solar cells with  $10^{16}$  cm<sup>-3</sup> base doping density, the carrier diffusion length is often less than  $100 \,\mu$ m. The thickness of our solar cell samples was  $200 \,\mu$ m, so a semi-infinite model can be adopted [12,14]. Therefore, the coefficient  $A_2$  in Eq. 3 must be zero, and the other three boundary conditions are

$$\begin{cases} D_{p} \left. \frac{dp(\omega, z)}{dz} \right|_{z=-d} = s_{f} p(\omega, z = -d) \\ p(0, 0) + p(\omega, 0) e^{i\omega t} = p_{0} \left( \exp \frac{V_{dc}}{V_{T}} - 1 \right) + p_{0} \frac{V_{ac}(\omega)}{V_{T}} \exp \frac{V_{dc}}{V_{T}} e^{i\omega t} \\ n(0, 0) + n(\omega, 0) e^{i\omega t} = n_{0} \left( \exp \frac{V_{dc}}{V_{T}} - 1 \right) + n_{0} \frac{V_{ac}(\omega)}{V_{T}} \exp \frac{V_{dc}}{V_{T}} e^{i\omega t}. \end{cases}$$
(4)

Here  $s_f$  is the front surface recombination velocity (FSRV),  $V_T = k_B T/q$  the thermal voltage,  $n_0$ ,  $p_0$  are, respectively, the equilibrium minority carrier densities in base/emitter, and the small ac photovoltage assumption is used to linearly expand the exponential function [12,13]. The current density through the junction can be expressed as

$$J = q \left[ D_{\rm p} \frac{\mathrm{d}p}{\mathrm{d}z} - D_{\rm n} \frac{\mathrm{d}n}{\mathrm{d}z} \right]_{z=0} = J_0^{\rm dc} \left( \exp \frac{V_{\rm dc}}{V_{\rm T}} - 1 \right) - J_g^{\rm dc} + J_0^{\rm ac} \frac{V_{\rm ac}}{V_{\rm T}} \exp \frac{V_{\rm dc}}{V_{\rm T}} - J_g^{\rm ac}$$
(5)

with

$$J_0 = q(n_0 D_{\rm n} \sigma_{\rm n} + p_0 D_{\rm p} \sigma_{\rm p} M) \tag{6a}$$

$$J_{\rm g} = q \left[ C_{\rm n} D_{\rm n} (\beta - \sigma_{\rm n}) e^{-\beta d} + C_{\rm p} D_{\rm p} \left( \sigma_{\rm p} N - \sigma_{\rm p} M e^{-\beta d} - \beta e^{-\beta d} \right) \right].$$
(6b)

Deringer

Here q is the elementary charge,  $J_0$  denotes the saturation current density (which also consists of both dc and ac parts), and M and N are FSRV-related quantities which have the forms

$$M = \frac{s_{\rm f} \cosh(\sigma_{\rm p} d) + D_{\rm p} \sigma_{\rm p} \sinh(\sigma_{\rm p} d)}{s_{\rm f} \sinh(\sigma_{\rm p} d) + D_{\rm p} \sigma_{\rm p} \cosh(\sigma_{\rm p} d)}$$
(6c)

$$N = \frac{s_{\rm f} + D_{\rm p}\beta}{s_{\rm f}\sinh(\sigma_{\rm p}d) + D_{\rm p}\sigma_{\rm p}\cosh(\sigma_{\rm p}d)}.$$
 (6d)

Similar to Eq. 3, the ac and dc currents are uniformly expressed by Eqs. 6a, b, with  $\sigma$  being real for dc and complex for ac excitation.

For a typical Si solar cell and one-sun illumination level, the shunt resistance is on the order of k $\Omega$  [9,10], much larger than the m $\Omega$ -order junction equivalent resistance (estimated by Re[ $V_{ac}(\omega)/J_{ac}(\omega)$ ]), while the junction capacitance is on the order of 10 µF, much smaller than the mF-order diffusion capacitance (estimated by Im[ $J_{ac}(\omega)/V_{ac}(\omega)$ ]/ $\omega$ ). Therefore, at open circuit, the effects of shunt resistance and junction capacitance can be neglected, and the current density through the junction, Eq. 5, can be treated as zero (no net current flow through the junction), which results in

$$\left(\exp\frac{V_{\rm dc}}{V_{\rm T}} - 1\right) = \frac{J_{\rm g}^{\rm dc}}{J_{\rm 0}^{\rm dc}}, \ \frac{V_{\rm ac}}{V_{\rm T}} \exp\frac{V_{\rm dc}}{V_{\rm T}} = \frac{J_{\rm g}^{\rm ac}}{J_{\rm 0}^{\rm ac}}.$$
(7)

One can find that the dc carrier density distribution is directly determined by the term  $\exp(V_{dc}/V_T) - 1$ , and the ac carrier density depth profile directly determined by the term  $(V_{ac}/V_T)[\exp(V_{dc}/V_T) - 1]$ , so Eq. 7 indicates that the carrier density distribution can be expressed by  $J_g/J_0$  for both ac and dc components.

Equation 6a, b show that both generation and saturation currents consist of contributions of electrons and holes (e.g., the first term on the RHS of Eq. 6a represents the contribution of electrons from the p-type base, while the second term denotes the contribution of holes from the n-type emitter). For the case that emitter doping (typically  $10^{19}$  cm<sup>-3</sup>) is much stronger than base doping (typically  $10^{16}$  cm<sup>-3</sup>), quantitative analysis shows that the electrons from the base dominate the generation and saturation currents through the junction, so Eq. 6a, b can be approximated as

$$\begin{cases} J_0 \approx q n_0 D_n \sigma_n \\ J_g \approx q C_n D_n (\beta - \sigma_n) e^{-\beta d} \end{cases} \Rightarrow \frac{J_g}{J_0} \approx \frac{C_n (\beta - \sigma_n) e^{-\beta d}}{n_0 \sigma_n} \tag{8}$$

Based on Eq. 8, the unknown coefficient  $B_2$  in Eq. 3b can be easily determined and the expression for the base carrier density depth profile is simplified to

$$n(\omega, z) \approx C_{\rm n} {\rm e}^{-\beta d} \left[ \frac{\beta}{\sigma_{\rm n}(\omega)} {\rm e}^{-\sigma_{\rm n}(\omega)z} - {\rm e}^{-\beta z} \right]$$
(9)

At one-sun injection level, the base excess minority carrier density was estimated using Eq. 9 to be on the order of  $10^{14}$  cm<sup>-3</sup>, two orders of magnitude smaller than the

 $10^{16}$  cm<sup>-3</sup> base doping, which is consistent with the earlier low injection assumption. The homodyne PCR signal generated in the base can be expressed as

$$S_{\text{HoPCR}}(\omega) \propto \int_0^\infty n(\omega, z) [p(\omega, z) + N_A] dz \approx N_A \int_0^\infty n(\omega, z) dz$$
 (10)

Incorporating Eq. 9 into Eq. 10, one obtains

$$S_{\text{HoPCR}}(\omega) \propto \int_0^\infty n(\omega, z) dz = \frac{N_0}{\tau_n^{-1} + i\omega}.$$
 (11)

Here  $N_0$  is the number of absorbed excitation photons which is equal to  $I_0(1 - R)\eta/2h\nu$ . It can be seen that the FSRV does not appear in the PCR/LIC expression due to the fact that the influence of the FSRV on the base carrier density depth profile is negligible (cf. Eqs. 8 and 9). In general, the PCR signal should consist of both the contributions from emitter and base. However, the contribution from the emitter can be omitted, because (1) the excess carrier density in the emitter is several orders of magnitude smaller than that in the base (same as the ratio of the doping difference) and (2) the thickness of the emitter (often 200 nm to 500 nm) is much less than that of the base, so the depth integral further helps the base PCR signal dominate over the emitter PCR signal. Therefore, it is concluded that the overall PCR signal is dominated by the base contribution to the signal.

The validity of the approximations leading to Eq. 11 which does not depend on the FSRV, but only on  $\tau_n$ , was tested using a full *p-n* junction PCR model (unpublished). The model showed that the influence of the FSRV depends on many parameters of the *p* and *n* layers of the solar cell; however, for the set of parameters used in the derivation of Eq. 11 with emitter doping density  $N_D = 10^{19} \text{ cm}^{-3}$  and base doping density  $N_A = 10^{16} \text{ cm}^{-3}$ , there is relatively small influence of the FSRV, consistent with the result of Eq. 11. It was also found that validity of Eq. 11 depends critically on the ratio  $N_D/N_A$ : decreasing the ratio leads to an increasing influence of FSRV.

#### **3** Experimental and Materials

The experimental PCR and LIC configuration has been described elsewhere [8]. The sample under investigation was an industrial mc-Si solar cell from Enfoton, Cyprus, with  $15.6 \times 15.6 \text{ cm}^2$  area and 200  $\mu$ m thickness. The average optical excitation intensity was about 0.8 suns (two 8-W lasers, square-wave modulated, homogenized and spread over a spot  $10 \times 10 \text{ cm}^2$ ). For heterodyne LIC, the two lasers were modulated by a two-channel function generator with a 10 Hz frequency difference, which was used as the reference frequency for lock-in measurements.

### **4 Results and Discussion**

Camera LIC amplitude images of the solar cell at open circuit are shown in Fig. 1. The image contrast is related to variations in CDW amplitude: large image ampli-



**Fig. 1** Camera LIC amplitude images of a mc-Si solar cell at open circuit: (a) 10 Hz homodyne; (b) 20 kHz heterodyne. Excitation intensity is  $\sim$ 0.8 sun

tude corresponds to high carrier density, i.e., long local carrier lifetime. The vertical equidistant dark bars are the current capturing electrode grids which act as drains of excess photocarriers. High amplitude locations are basically at mid-distance between adjacent grids where carrier densities are expected to be high. To quantitatively investigate the PCR/LIC frequency responses of the cell, single-element InGaAs detector homodyne frequency scans of three selected locations of the cell (A, B, C, shown in Fig. 1a) were performed and the corresponding results are shown in Fig. 2. Using Eq. 11 to best fit the frequency-dependent data, the base minority carrier lifetimes at the three locations were found to be  $18.4 \,\mu s$ ,  $17.0 \,\mu s$ , and  $12.7 \,\mu s$ , respectively.

By comparison between the low-frequency homodyne image at 10 Hz (Fig. 1a) and the high-frequency heterodyne image at 20 kHz (Fig. 1b), it is clearly seen that the contrast of two vertical regions (at the center and the right side of the image) against the surrounding area is reversed. To account for this contrast inversion phenomenon,

Fig. 2 Single-element InGaAs detector homodyne frequency scans of the three selected locations on the cell (shown on Fig. 1a): (a) amplitude-frequency dependences; (b) phase-frequency dependences. Also shown are the theoretical best fits



two facts need to be considered. The first fact can be found in the diffusive character of photocarriers which are controlled by the frequency-dependent ac diffusion length. The fitting results show that the lifetimes at locations A and B are longer than that at location C, so at high frequencies a heavier amplitude damping will occur in longlifetime regions than that in poor quality areas where the local carrier lifetime is relatively shorter and the damping rate versus frequency is thus slower.

The second fact is related to a different nature of homodyne and heterodyne signals. The heterodyne signal has the form

$$S_{\text{HePCR}}(t) \propto \int_{0}^{\infty} \frac{\left[2n(0,z) + n(\omega_{1},z)e^{i\omega_{1}t} + n(\omega_{2},z)e^{i\omega_{2}t}\right] \times}{\left[2p(0,z) + p(\omega_{1},z)e^{i\omega_{1}t} + p(\omega_{2},z)e^{i\omega_{2}t} + N_{\text{A}}\right] \mathrm{d}z.}$$
(12)

Here the terms within the first square brackets of the integrand denote the photogenerated minority carrier densities (including one dc term and two ac components with different frequencies), while the terms within the second square brackets correspond to the majority carrier densities (including the photogenerated carrier densities and the doping density). Under quasi-neutral approximation, photogenerated minority carrier density  $n(\omega, z)$  is equal to photogenerated majority carrier density  $p(\omega, z)$ . Among the terms in the integrand of Eq. 12, only the two products of the ac components with



different frequencies contribute to the beat heterodyne signal. Unlike the homodyne signal expressed by Eq. 10, the heterodyne signal has a higher order of dependence on the ac carrier density, which makes the damping rate of heterodyne signals versus frequency higher. This can be validated by analyzing the homodyne and heterodyne amplitude data shown in Figs. 2a and 3: it can be seen that the ratios of the heterodyne amplitude data at the lowest and highest frequencies are consistently larger than those of the homodyne amplitude data.

Using Eq. 12 to best fit the heterodyne amplitude-frequency dependencies at the three locations, the results are shown in Fig. 3. The contrast inversion observed between Fig. 1a and b is now clearly seen. As frequency increases, the CDW from the highquality region becomes heavily damped, while the frequency-mixing nature of the heterodyne signal further amplifies the damping. The signal from the electronically poor material region becomes less damped due to the shorter lifetimes involved, so that the amplitude curves intersect, with the poor region amplitude extending beyond and above that from the good region at frequencies above the crossing value. The base minority carrier lifetimes at the three locations extracted from the camera heterodyne data were found to be  $4.3 \,\mu$ s,  $3.8 \,\mu$ s, and  $2.3 \,\mu$ s, respectively, shorter than those obtained from the homodyne data. The reason for the difference in values is still under investigation. One possible interpretation is that by comparing Eq. 12 to 10, one can see that both free-carrier recombination, expressed by  $n(\omega) p(\omega)$ , and radiative recombination of donor- and acceptor-bound carriers, expressed by  $N_A n(\omega)$ , contribute to homodyne signals, while in the heterodyne mode, only free-carrier recombination, expressed by  $n(\omega_1)p(\omega_2)$ , can generate the beat frequency component by frequency mixing. This inference indicates that homodyne and heterodyne PCR may measure different carrier recombination lifetimes.

## **5** Conclusions

A simplified solar cell PCR/LIC model was presented for the case of open circuit condition and spread beam illumination. It was shown that the base carrier density depth profile is closely related to the ratio of the generation and the saturation currents, and the base minority carriers play dominant roles over their emitter counterparts in

both generation and saturation currents through the junction. It was shown that that PCR/LIC signals are sensitive to the base minority carrier lifetime and not sensitive to the FSRV under typical mc-Si solar cell parameters. Heterodyne LIC images of a mc-Si solar cell up to 20 kHz were obtained, and base minority carrier lifetimes of the cell at selected locations were extracted from both homodyne single-element detector data and heterodyne camera images. Recombination lifetimes were found to be longer when measured from the heterodyne PCR frequency response than from homodyne PCR.

**Acknowledgments** The authors are grateful to the Canada Research Chairs Program and to NSERC for a Discovery grant to A. Mandelis who also gratefully acknowledges the Chinese Recruitment Program of Global Experts (Thousand Talents). QMS gratefully acknowledges the NNSFC (Grant No. 61574030) and the project sponsored by OATF, UESTC.

# References

- 1. A. Mandelis, J. Batista, D. Shaughnessy, Phys. Rev. B 67, 205208 (2003)
- 2. J. Batista, A. Mandelis, D. Shaughnessy, Appl. Phys. Lett. 82, 4077 (2004)
- 3. B.C. Li, D. Shaughnessy, A. Mandelis, J. Appl. Phys. 97, 023701 (2005)
- 4. J. Batista, A. Mandelis, D. Shaughnessy, B.C. Li, Appl. Phys. Lett. 85, 1713 (2004)
- 5. D. Shaughnessy, A. Mandelis, J. Electrochem. Soc. 153, G283 (2006)
- 6. A. Melnikov, A. Mandelis, J. Tolev, P. Chen, S. Huq, J. Appl. Phys. 107, 114513 (2010)
- 7. Q. Sun, A. Melnikov, A. Mandelis, Appl. Phys. Lett. 101, 242107 (2012)
- 8. A. Melnikov, P. Chen, Y. Zhang, A. Mandelis, Int. J. Thermophys. 33, 2095 (2012)
- 9. J.Y. Liu, A. Melnikov, A. Mandelis, J. Appl. Phys. 114, 104509 (2013)
- 10. J.Y. Liu, A. Melnikov, A. Mandelis, Phys. Status Solidi A 210, 2135 (2013)
- 11. Q. Sun, A. Melnikov, A. Mandelis, Int. J. Thermophys. 36, 1274 (2015)
- 12. A. Mandelis, J. Appl. Phys. 66, 5572 (1989)
- 13. J.Y. Liu, A. Mandelis, L. Qin, Y. Wang, P. Song, A. Melnikov (in preparation)
- 14. J.P. McKelvey, in *Solid Slate and Semiconductor Physics*, Chap. 13.6 (Harper and Row, New York, 1966)